Large Deviation Behavior for The Longest Head Run in IID Bernoulli Sequence

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Abstract: This paper discusses large deviation behavior of the longest perfect head run in i.i.d. Bernoulli sequence. Let $Z_1, Z_2, \ldots$ be an i.i.d. sequence with $P(Z_i = 1) = 1 - P(Z_i = 0) = p = 1 - q$ and S_N be the length of the longest consecutive run of 1's within the first N tosses. The famous Erdös-Rényi law tells that $S_N / \ln N \to \xi(p) := [-\ln p]^{-1}$ almost surely as $N \to \infty$. It is proved in this paper that, while $P[S_N / \ln N \geq \xi(p) + x]$ decays like $N^{-x/\xi(p)}$ for each $x > 0$, $P[S_N / \ln N \leq \xi(p) - x]$ decays like $\exp\{-O(N^{x/\xi(p)})\}$ for $0 < x < \xi(p)$.

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