Perturbation by Non-Local Operators

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Abstract: Suppose that $d \geq 1$ and $0 < \beta < \alpha < 2$. We establish the existence and uniqueness of the fundamental solution $q^b(t,x,y)$ to non-local operators $\mathcal{L}^b = \Delta^{\alpha/2} + \mathcal{S}^b$, where

$$\mathcal{S}^b f(x) := \mathcal{A}(d,-\beta) \int_{\mathbb{R}^d} \left(f(x+z) - f(x) - \langle \nabla f(x), z \mathbb{1}_{\{|z| \leq 1\}} \rangle \right) \frac{b(x,z)}{|z|^{d+\beta}} dz$$

and $b(x,z)$ is a bounded measurable function on $\mathbb{R}^d \times \mathbb{R}^d$ with $b(x,z) = b(x,-z)$ for $x, z \in \mathbb{R}^d$. Here $\mathcal{A}(d,-\beta)$ is a normalizing constant. We show that if $b \geq 0$, then $q^b(t,x,y)$ is a strictly positive continuous function and it uniquely determines a conservative Feller process X^b that has strong Feller property. The Feller process X^b is the unique solution to the martingale problem of $(\mathcal{L}^b, \mathcal{S}(\mathbb{R}^d))$, where $\mathcal{S}(\mathbb{R}^d)$ is the space of tempered functions on $\mathbb{R}^d$. Furthermore, sharp two-sided estimates on $q^b(t,x,y)$ is derived.

This is a joint work with Professor Z.-Q. Chen.

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