Functional Inequalities for Stable-Like Dirichlet Forms

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Abstract: Let $V \in C^2(\mathrm{R}^d)$ such that $\mu_V(dx) := e^{-V(x)}\,dx$ is a probability measure, and let $\alpha \in (0,2)$. Explicit criteria are presented for the α-stable-like Dirichlet form

$$D_{\alpha,V}(f,f) := \int_{\mathrm{R}^d\times\mathrm{R}^d} \frac{|f(x)-f(y)|^2}{|x-y|^{d+\alpha}}\,dy\,\mu_V(dx)$$

to satisfy Poincaré-type (i.e., Poincaré, weak Poincaré and super Poincaré) inequalities. As applications, sharp functional inequalities are derived for the Dirichlet form with V having some typical growths. Finally, the main result of [1] on the Poincaré inequality is strengthened.

References

[1] C. Mouhot, E. Russ, Y. Sire, *Fractional Poincaré inequalities for general measures,* J. Math. Pures Appl. **95** (2011), 72–84.