

Integration by Parts Formula and Shift Harnack Inequality for Stochastic Equations

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Abstract: A new coupling argument is introduced to establish Driver's integration by parts formula and shift Harnack inequality. Unlike known coupling methods where two marginal processes with different starting points are constructed to move together as soon as possible, for the new-type coupling the two marginal processes start from the same point but their difference is aimed to reach a fixed quantity at a given time. Besides the integration by parts formula, the new coupling method is also efficient to imply the shift Harnack inequality. Differently from known Harnack inequalities where the values of a reference function at different points are compared, in the shift Harnack inequality the reference function, rather than the initial point, is shifted. A number of applications of the integration by parts and shift Harnack inequality are presented. The general results are illustrated by some concrete models including the stochastic Hamiltonian system where the associated diffusion process can be highly degenerate, delayed SDEs, and semi-linear SPDEs.