

# Generalized Stochastic Flow Associated to the Itô SDE with Partially Sobolev Coefficients and Applications

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KEY WORDS: Di Perna–Lions theory, stochastic differential equation, density estimate, weak differentiability, maximal function

MATHEMATICAL SUBJECT CLASSIFICATION: Primary 60H10; secondary 42B25

**Abstract:** We consider the Itô SDEs with partially Sobolev coefficients. Assuming the exponential integrability of the negative part of the divergence of the drift coefficient and the partial gradient of the diffusion coefficient with respect to the generalized Cauchy measure, we show the existence, uniqueness and stability of generalized stochastic flows associated to such equations. As an application, we prove the weak differentiability in the sense of measure of the stochastic flow generated by the Itô SDE with Sobolev coefficients.

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