Convergence to equilibrium of Markov processes and functional inequalities via Lyapunov conditions

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(with D. Bakry, F. Barthe, P. Cattiaux, R. Douc, N. Gozlan, C. Roberto, F.Y. Wang, X. Wang, L. Wu)

Introduction

Let $(X_t)_{t\geq 0}$ be a continuous time Markov process (say a diffusion for simplicity) with

- P_t its associated semigroup,
- $\mathcal{L}$ its generator,
- μ its invariant probability measure,
- $\mathcal{E}(f,g) = \int -f\,L\,g\,d\mu$ its associated Dirichlet form.

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Our goal:

"quantify the decay to 0 of $d(P_t, \mu)$"

for some distance d with easy to verify conditions.

Running Example

(X_t) satisfies the stochastic differential equation

$$dX_t = \sqrt{2}\, dB_t - \nabla V(X_t)dt$$

and has as generator

$$\mathcal{L} = \Delta - \nabla V.\nabla$$

with invariant measure

$$d\mu(x) = e^{-V(x)}dx$$

and Dirichlet form

$$\mathcal{E}(f,f) = \int |\nabla f|^2\, d\mu.$$

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- Functional inequalities and d is linked to the L^2 norm (Poincaré) or the entropy (logarithmic Sobolev).

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and local conditions.

Lyapunov conditions

The prototype of Lyapunov condition is: find $W \geq 1$, a set C, $b > 0$ and positive function φ

$$\mathcal{L}W \leq -\varphi \times W + b1_C.$$

It has been used since a long time to study speed of convergence to equilibrium but often without explicit constant.

Examples

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but with another choice

$$W(x) = e^{a|x|^2}, \qquad \mathcal{L}W = \left(2an + 4a\left(a - \frac{1}{2}\right)|x|^2\right)W(x) \leq -\lambda|x|^2 W(x) + b1_{\{|x| \leq R\}}$$

Examples continued

– Exponential type process: $\mathcal{L} = \Delta - \frac{x}{|x|}.\nabla$.
Choose $a < 1$

$$W(w) = e^{a|x|}, \qquad \mathcal{L}W \leq -c\,W(x) + b1_{\{|x| \leq R\}}$$

– Cauchy type process: $\mathcal{L} = \Delta - (n+\alpha)\frac{\nabla V}{V}.\nabla$ and V convex.

choose now $2 < k < \alpha(1-\varepsilon) + n\varepsilon + 2$ for ε sufficiently small then

$$W(x) = 1 + |x|^k, \qquad \mathcal{L}W \leq -c\,(W(x))^{\frac{k-2}{k}} + b1_{\{|x| \leq R\}}$$

Functional Inequalities

Let us quickly illustrate the method :

Suppose that the following Poincaré inequality is verified

$$\textbf{(PI)} \qquad \mathrm{Var}_\mu(f) := \mu(f^2) - \mu(f)^2 \le C_p\, \mathcal{E}(f,f)$$

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then

$$\begin{aligned} \frac{d}{dt}\mathrm{Var}_\mu(P_t f) &= 2\int P_t f\, L\, P_t f\, d\mu \\ &\leq -\frac{2}{C_p}\mathrm{Var}_\mu(P_t f) \end{aligned}$$

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$$Var_\mu(P_t f) \leq e^{-\frac{2}{C_p}t}\,\mathrm{Var}_\mu(f)$$

(In fact equivalent to Poincaré inequality)

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– weak Poincaré inequality : $\forall s > 0$

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– logarithmic Sobolev inequality

$$\textbf{(LSI)} \qquad \forall s > 0 \qquad \mathrm{Ent}_\mu(f) := \mu\left(f \log \frac{f}{\mu(f)}\right) \leq 2C_l\,\mathcal{E}(f, \log f)$$

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$$\mathrm{Ent}_\mu(P_t f) \leq e^{-\frac{2}{C_l}t}\,\mathrm{Ent}_\mu(f)$$

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We will focus on a recent approach based on Lyapunov conditions.

Poincaré inequality

Theorem

Let $\mathcal{L} = \Delta - \nabla V . \nabla$. *Suppose that there exists* $W \geq 1$, $\lambda, b > 0$ *and* $R > 0$ *such that*

$$\mathcal{L}W \leq -\lambda W + b 1_{B(0,R)}$$

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and a local Poincaré inequality : for f *such that* $\mu(f1_{B(0,R)}) = 0$

$$\int_{B(0,R)} f^2 d\mu \leq \kappa_R \int |\nabla f|^2 d\mu$$

then

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then

$$Var_\mu(f) \leq \frac{1}{\lambda}(1 + b\kappa_R) \int |\nabla f|^2 d\mu$$

Remark

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The Lyapunov condition is verified for example if

- $x.\nabla V \geq \alpha|x|$ *for some positive* α *outside a ball;*
- *or* $a|\nabla V|^2 - \Delta V \geq c$ *for* $0 < a < 1$ *and positive* c *outside a ball.*

In particular, if V *is convex then the first condition is verified and thus a Poincaré inequality holds (recovering a result of Bobkov).*

Proof

Remark first

$$\begin{aligned}
\int f^2 \frac{-\mathcal{L}W}{W} d\mu &= \int \nabla \left(\frac{f^2}{W}\right).\nabla W \, d\mu \\
&= 2\int \frac{f}{W}\nabla f.\nabla W \, d\mu - \int \frac{f^2}{W^2}|\nabla W|^2 d\mu \\
&= \int |\nabla f|^2 d\mu - \int |\nabla f - (f/W)\nabla W|^2 d\mu \\
&\leq \int |\nabla f|^2 d\mu.
\end{aligned}$$

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or a large deviations argument!

So that with $c = \mu(f 1_{B(0,R)})$, the Lyapunov condition rewritted

$$1 \leq -\frac{1}{\lambda}\frac{\mathcal{L}W}{W} + \frac{b}{\lambda} 1_{B(0,R)}$$

and local Poincaré inequality

$$\begin{aligned} \mathrm{Var}_\mu(f) &\leq \int (f-c)^2 d\mu \\ &\leq \frac{1}{\lambda}\int (f-c)^2 \frac{-\mathcal{L}W}{W} d\mu + \frac{b}{\lambda}\int_{B(0,R)} (f-c)^2 d\mu \\ &\leq \frac{1}{\lambda}(1 + b\kappa_R) \int |\nabla f|^2 d\mu \end{aligned}$$

By a slight modification of the argument, it extends to weighted and weak Poincaré inequality

Theorem

Let $\mathcal{L} = \Delta - \nabla V.\nabla$. *Suppose that there exists* $W \geq 1$, *sublinear* φ, $b > 0$ *and* $R > 0$ *such that*

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$$Var_\mu(f) \leq \max\left(1, \frac{b\kappa_R}{\varphi(1)}\right) \int (1 + \varphi'(W)^{-1})|\nabla f|^2 d\mu$$

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and denoting $G(s) = \inf\{u; \mu(\varphi(W) < uW) > s\}$ *then*

$$Var_\mu(f) \leq (1 + b\kappa_R) G(s)^{-1} \int |\nabla f|^2 d\mu + 2s\|f\|_\infty^2$$

Remark

For Cauchy type process : the Lyapunov condition is such that $\varphi'(W) = |x|^2$ *and* $G(s)^{-1} = s^{-\frac{2}{\alpha}}$ *which are optimal in dimension one (see Barthe-Cattiaux-Roberto) and enable to recover recent results of Bobkov-Ledoux.*

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Remark

Note that weighted Poincaré inequality enables us to find easily a reversible diffusion exponentially convergent in L^2 *for a given subexponential measure satisfying a Lyapunov condition: take* $\omega = (1 + \varphi'(W)^{-1})$ *and use the reversible diffusion*

$$L^\omega = \omega\Delta + (\nabla\omega - \omega\nabla V).\nabla.$$

Theorem

Let $\mathcal{L} = \Delta - \nabla V.\nabla$ *and* $d\mu = e^{-V}dx$. *Suppose that there exists* $W \geq 1$, *some point* x_0, $b > 0$ *such that*

$$\mathcal{L}W \leq -c\, d^2(x, x_0) \times W + b$$

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then the following transportation inequality holds : for all probability measure ν

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$$W_2^2(\nu, \mu) \leq C\, Ent_\mu\left(\frac{d\nu}{d\mu}\right)$$

and if we suppose moreover $Hess(V) + Ric \geq K\, Id$ then the logarithmic Sobolev inequality holds

$$Ent_\mu(f^2) \leq C \int |\nabla f|^2 d\mu$$

Remark

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We have a powerful tool to get functional inequalities and thus various rates of convergence to equilibrium but...

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so let's try another approach.

Coupling

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find some random time T and a construction of X_t and Y_t both of the same law given by P_t starting from x and y which coincides after time T, so that

$$\|P_t(x,\cdot) - P_t(y,\cdot)\|_{TV} \leq \mathbb{P}(T > t)$$

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We will also use Lyapunov condition and a minorization condition.

Coupling construction

Suppose that for some set C, some $t^* > 0$, there exists $\varepsilon > 0$ such that

$$(MC) \qquad \forall x \in C, \qquad P_{t^*}(x, \cdot) \geq \varepsilon \nu(\cdot)$$

Construction of (X_t, Y_t)

1. $X_0 = x,\ Y_0 = y$
2. Let $t_0 = \inf\{t;\ (X_t, Y_t) \in C \times C\}$ and $t_n = \inf\{t \geq t_{n-1} + t^*;\ (X_t, Y_t) \in C \times C\}$. Then proceed at each t_i
   - If not coupled, with probability ε, $X_{t_i+t^*} = Y_{t_i+t^*} = Z$ with $Z \sim \nu$ and declare to have coupled and $T = t_i + t^*$!.
   - if not coupled, with probability $1 - \varepsilon$, simulate conditionally independently $X_{t_i+t^*}$ and $Y_{t_i+t^*}$ with the residual kernel and go on.

We have thus to control integrability of entrance time to some set C : $\tau_C(t^*) = \inf\{t \geq t^*, X_t \in C\}$

Theorem

Suppose that there exists $W \geq 1$, $\delta > 0$, $b > 0$ *such that*

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$$\forall x \notin C, \, \mathbb{E}_x(e^{\delta \tau_C(0)}) \leq W(x)$$

and if exists $W \geq 1$, *sublinear concave* φ, $b > 0$ *such that*

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$$\forall x \notin C,\ \mathbb{E}_x\left(H_\varphi^{-1}(\tau_C(t^*)\right) \leq W(x) + c_{b,\varphi,\theta^*}$$

where $H_\varphi(u) = \int_1^u \frac{1}{\varphi} ds$.

Proof

By Itô's formula $e^{\delta(t\wedge\tau_C(0))}W(X_{t\wedge\tau_C(0)})$ is a local supermartingale so that

$$\mathbb{E}(W(X_0)) \geq \mathbb{E}(e^{\delta\tau_C(0)}W(X_{\tau_{C(0)}})) \geq \mathbb{E}(e^{\delta\tau_C(0)}).$$

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The subexponential case is much more involved.

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then for an explicit $\rho = \rho(\varepsilon, C, t^*, W) < 1$ $K > 1$,

$$\|P_t(x,\cdot) - P_t(y,\cdot)\|_{TV} \leq K\rho^t \left(W(x) + W(y)\right)$$

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then for an explicit $K = K(\varepsilon, C, \varphi, W)$ *and* $\lambda(C, \varphi, W)$

$$\|P_t(x,\cdot) - P_t(y,\cdot)\|_{TV} \leq \frac{K}{H_\varphi^{-1}(\lambda(t - t^*))} \left(W(x) + W(y)\right)$$

Remark

- *The coupling approach in the simple reversible setting may be compared to the functional inequality approach.*
- *It enables us to consider non reversible models such as kinetic Fokker-Planck equation*

$$\begin{aligned} d\,x_t &= v_t\,dt \\ d\,v_t &= \sqrt{2}dB_t - \nabla V(x_t)dt - cv_t dt \end{aligned}$$

under various conditions on V.

- *The difficulty is of course in the estimation in the minorization condition.*

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