

CRITICAL BROWNIAN SHEET DOES NOT HAVE DOUBLE POINTS

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Abstract: The Brownian sheet is a multiparameter extension of Brownian motion and plays important roles in probability theory and stochastic partial differential equations. Sample functions of the Brownian sheet generate many interesting random fractals and have been studied by many authors.

This talk is concerned with potential theory of the Brownian sheet and its fractal geometric properties. We show that an N -parameter Brownian sheet in \mathbf{R}^d has double points if and only if $2(d - 2N) < d$. In particular, in the critical case where $2(d - 2N) = d$, Brownian sheet does not have double points. This answers an old problem in the folklore of the subject. We also discuss some of the geometric consequences of the mentioned decoupling, and establish a partial result concerning k -multiple points in the critical case $k(d - 2N) = d$.

This talk is based on joint work with Robert Dalang, Davar Khoshnevisan, Eulalia Nualart and Dongsheng Wu.

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