BURGERS-KPZ TYPE PDES CHARACTERISING THE PATH-INDEPENDENT PROPERTY OF THE DENSITY OF THE GIRSANOV TRANSFORMATION FOR SDES

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Abstract: Let X_t solve the (multidimensional) Itô's SDEs on $\mathbf{R}^d$

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dB_t$$

where $b : [0, \infty) \times \mathbf{R}^d \to \mathbf{R}^d$ is smooth in its two arguments, $\sigma : [0, \infty) \times \mathbf{R}^d \to \mathbf{R}^d \otimes \mathbf{R}^d$ is smooth with $\sigma(t, x)$ being invertible for all $(t, x) \in [0, \infty) \times \mathbf{R}^d$, B_t is d-dimensional Brownian motion. It is shown that, associated to a Girsanov transformation, the stochastic process

$$\int_0^t \langle (\sigma^{-1}b)(s, X_s), dB_t \rangle + \frac{1}{2}\int_0^t |\sigma^{-1}b|^2(s, X_s)ds$$

is a function of the arguments t and X_t (i.e., path-independent) if and only if $b = \sigma\sigma^*\nabla v$ for some scalar function $v : [0, \infty) \times \mathbf{R}^d \to \mathbf{R}$ satisfying the time-reversed Burgers-KPZ type equation

$$\frac{\partial}{\partial t} v(t, x) = -\frac{1}{2}\left[\left(Tr(\sigma\sigma^*\nabla^2 v) \right)(t, x) + |\sigma^*\nabla v|^2(t, x) \right].$$

The assertion also holds on a connected complete differential manifold.

References

[1] J.-L. Wu, W. Yang:On stochastic differential equations and generalised Burgers equation. Submitted.

[2] A. Truman, F.-Y. Wang, J.-L. Wu, W. Yang:A Burgers-KPZ type parabolic equation for the path-independence of the density of the Girsanov transformation. Submitted.