

CENTRAL LIMIT THEOREM FOR THE BROWNIAN LOCAL TIME IN L_p SPACE

Yaozhong HU *University of Kansas, USA*, E-mail: hu@math.ku.edu

KEY WORDS: Brownian motion, local time, central limit theorem, Malliavin calculus, Clark-Ocone formula, Ray-Knight theorem, Tanaka formula

MATHEMATICAL SUBJECT CLASSIFICATION: 60H15

Abstract: Let $(B_t, t \geq 0)$ be the Brownian motion and L_t^x be the local time. In this paper we use Malliavin calculus to give a new proof of the following theorem: For each fixed $t > 0$

$$\frac{1}{h^2} \int_{\mathbf{R}} (L_t^{x+h} - L_t^x)^3 dx \longrightarrow 8\sqrt{3} \left(\int_{\mathbf{R}} (L_t^x)^3 dx \right)^{\frac{1}{2}} \eta$$

as h tends to zero, where η is a normal random variable with mean zero and variance one that is independent of B . The techniques are based on ingredients: The Clark-Ocone formula of the predictable representation of a random variable and an asymptotic version of Ray-Knight's theorem.

References

- [1] Barlow, M. T.; Yor, M. Semimartingale inequalities via the Garsia-Rodemich-Rumsey lemma, and applications to local times. *J. Funct. Anal.* **49** (1982), no. 2, 198–229.
- [2] Borodin, A. N. Brownian local time. *Russian Math. Surveys* **44** (1989), 1–51.
- [3] Chen, X., Li, W., Marcus, M. B. and Rosen, J: A CLT for the L^2 modulus of continuity of Brownian local time. Preprint.
- [4] Nualart, D.; Hu, Y. Stochastic integral representation of the L^2 modulus of Brownian local time and a central limit theorem. Preprint.
- [5] M. B. Marcus and J. Rosen, L_p moduli of continuity of Gaussian processes and local times of symmetric Lévy processes, *Annals of Probab.* To appear.
- [6] Nualart, D. *The Malliavin Calculus and Related Topics*. Second edition. Springer Verlag, Berlin, 2006.
- [7] Ocone, D. Malliavin calculus and stochastic integral representation of diffusion processes. *Stochastics* **12** (1984), 161–185.
- [8] Pitman, J.; Yor, M. Asymptotic laws of planar Brownian motion. *Ann. Probab.* **14** (1986), no. 3, 733–779.
- [9] Revuz, D.; Yor, M. *Continuous martingales and Brownian motion*. Third edition. Springer-Verlag, Berlin, 1999.
- [10] Rogers, L. C. G.; Walsh, J. B. The exact 4/3-variation of a process arising from Brownian motion. *Stochastics Stochastics Rep.* **51** (1994), no. 3-4, 267–291.
- [11] Rosen, L. Derivatives of self-intersection local times. Preprint
- [12] Rosen, J. A CLT for the third integrated moment of Brownian local time. Preprint.