A CONTINUUM-TREE-VALUED MARKOV PROCESS

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Abstract: We present a construction of a Lévy continuum random tree (CRT) (see Duquesne and Le Gall) associated with a super-critical continuous state branching process using the so-called exploration process and a Girsanov theorem. We also extend the pruning procedure developped by Abraham, Delmas and Voisin to this super-critical case. Let ψ be a critical branching mechanism. We set $\psi_\theta(\cdot) = \psi(\cdot + \theta) - \psi(\theta)$. Let $\Theta = (\theta_\infty, +\infty)$ or $\Theta = [\theta_\infty, +\infty)$ be the set of values of θ for which ψ_θ is a conservative branching mechanism. The pruning procedure allows to construct a decreasing Lévy-CRT-valued Markov process $(\mathcal{T}_\theta, \theta \in \Theta)$, such that $\mathcal{T}_\theta$ has branching mechanism ψ_θ. It is sub-critical if $\theta > 0$ and super-critical if $\theta < 0$. We then consider the explosion time A of the CRT: the smallest (negative) time θ for which the continuous state branching process (CB) associated with $\mathcal{T}_\theta$ has finite total mass. We describe the law of A as well as the distribution of the CRT just after this explosion time. The CRT just after explosion can be seen as a CRT conditioned not to be extinct which is pruned with an independent intensity related to A. We also study the evolution of the CRT-valued process after the explosion time. This extends results from Aldous and Pitman on Galton-Watson trees, see also Abraham, Delmas and He for an other approach on Galton-Watson trees. For the particular case of the quadratic branching mechanism, we show that after explosion the total mass of the CB behaves like the inverse of a stable subordinator with index 1/2. This result is related to the size of the tagged fragment for the fragmentation of Aldous' CRT.

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