

LIMIT THEOREMS FOR FUNCTIONS OF MARGINAL QUANTILES

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Abstract: Multivariate distributions are explored using the joint distributions of marginal sample quantiles. Limit theory for the mean of a function of order statistics is presented. The results include a multivariate central limit theorem and a strong law of large numbers. A result similar to Bahadur's representation of quantiles is established for the mean of a function of the marginal quantiles. In particular, it is shown that

$$\sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \phi \left(X_{n:i}^{(1)}, \dots, X_{n:i}^{(d)} \right) - \bar{\gamma} \right) = \frac{1}{\sqrt{n}} \sum_{i=1}^n Z_{n,i} + o_P(1)$$

as $n \rightarrow \infty$, where $\bar{\gamma}$ is a constant and $Z_{n,i}$ are *i.i.d.* random variables for each n . This leads to the central limit theorem. Weak convergence to a Gaussian process using equicontinuity of functions is indicated. The results are established under very general conditions. These conditions are shown to be satisfied in many commonly occurring situations.