ASYMPTOTIC EXPANSION WITH DOUBLE LAYERS OF FORWARD EQUATIONS OF MARKOV CHAINS

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Abstract: For a Markov chain on a finite state space $\{1, 2, \cdots, m\}$ with (possibly inhomogeneous) transition rate matrix

$$Q^\epsilon(t) = 1/\epsilon^2 \cdot Q_2(t) + 1/\epsilon \cdot Q_1(t) + Q_0(t),$$

we consider its Forward equation

$$\partial p/\partial t = p(t)Q^\epsilon(t)$$
$$p^\epsilon(0) = p_0$$

where $p(t) = (p(X_t = 1), p(X_t = 2), \cdots, p(X_t = m))$ is the probability distribution of X_t and p_0 is any given initial distribution.

We shall show that there exist functions $f^{(i)}$ (regular part), $h^{(i)}$ (first layer) and $g^{(i)}$ (double layer) such that

$$\sup_{0\le t\le T} \Big| p^\epsilon - \Big(\sum_{i=1}^{n} \epsilon^i f^{(i)}(t) + \sum_{i=1}^{n} \epsilon^i h^{(i)}(t/\epsilon) + \sum_{i=1}^{n} \epsilon^i g^{(i)}(t/\epsilon^2) \Big) \Big| = O(\epsilon^{n+1}).$$

Moreover, we want to have that $h^{(i)}(\tau) \le K \exp(-\tau\gamma)$ and $g^{(i)}(\tau) \le K \exp(-\tau\gamma)$ for some positive constants τ and γ for each $h^{(i)}$ and $g^{(i)}$.

Previous work [1] considered the asymptotic expansion of Forward equation with two time scales and we generalize their results to three scales using the same matched expansion methods. However, some new phenomena emerge and we shall discuss their probabilistic interpretation.

References

[1] George Yin and Qing, Zhang (1997). Continuous-Time Markov Chains and Applications, A Singular Perturbation Approach Springer.