SEMIGROUPS THAT PRESERVE A CONVEX SET IN A BANACH SPACE

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Abstract:

Let B be a Banach space and $\{T_t\}$ be a semigroup in B. Suppose we are given a convex set C in B. We are interested in when the semigroup preserves the convex set C, i.e., $T_tC \subseteq C$ for any $t \geq 0$. This kind of issue was discussed by Brezis-Pazy in Hilbert space case and we extend it to Banach space case.

This theorem covers the following cases:

1. positivity preserving
2. Markov property
3. L^1 contraction
4. excessive function
5. invariant function

In each case, the necessary and sufficient conditions are rather well-known but the point of the talk is that they can be treated in a unified way. We also discussed Hilber space case. In that case, the conditions are described in terms of bilinear forms.

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