QUADRATIC COVARIATION AND ITÔ'S FORMULA FOR A BIFRACTIONAL BROWNIAN MOTION

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KEY WORDS: Fractional Brownian motion, Wick-Itô stochastic integral, fractional Itô formula, Girsanov thoerem for fractional Brownian motion, fractional Clark-Ocone theorem, option pricing

Abstract: Let $B^{H,K}$ be a bifractional Brownian motion $B^{H,K}$ with indices H, K such that $2HK = 1$, and denote by $\mathscr{L}^{H,K}(x,t)$ its local time process. In this paper, we study the integral with respect to $\mathscr{L}^{H,K}(x,t)$ of the form

$$\int_{\mathbb{R}}\int_0^t f(x,s)\mathscr{L}^{H,K}(dx,ds),$$

and the quadratic covariation $\left[f(B^{H,K},\cdot), B^{H,K}\right]$ of $f(B^{H,K},\cdot)$ and $B^{H,K}$, where $(x,t)\mapsto f(x,t)$ is a determinate function. We construct Banach space $\mathscr{H}_*$ such that the above integral and quadratic covariation exist in L^1 for all $f\in\mathscr{H}_*$, and we have

$$\int_{\mathbb{R}}\int_0^t f(x,s)\mathscr{L}^{H,K}(dx,ds) = -2^{K-1}\left[f(B^{H,K},\cdot), B^{H,K}\right]_t$$

for all $t \geq 0$. As an application we derive the Itô formula of the form

$$F(B_t^{H,K},t) = F(0,0) + \int_0^t F_x(B_s^{H,K},s)dB_s^{H,K} + \int_0^t F_t(B_s^{H,K},s)ds \\ -\frac{1}{2}\int_0^t\int_{\mathbb{R}} F_x(x,s)\mathscr{L}^{H,K}(dx,ds),$$

where $F_x = \frac{\partial F}{\partial x}\in\mathscr{H}_*,\ F_t = \frac{\partial F}{\partial t}$.