DECOMPOSITIONS AND STRUCTURAL ANALYSIS OF STATIONARY INFINITELY DIVISIBLE PROCESSES

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Abstract: Path Lévy measures for many classes of infinitely divisible processes $\mathbf{X} = \{X_n\}_{n\in\mathbf{Z}}$ can be characterized as the product convolutions of a certain 'root' Lévy measure ρ on $\mathbf{R}$ and measures on $\mathbf{R}^{\mathbf{Z}}$. For example, selfdecomposable processes indexed by $\mathbf{Z}$ have their path Lévy measures of the form

$$\nu(A) = \int_0^\infty \eta(x^{-1}A)\,\rho(dx), \qquad A \subset \mathbf{R}^{\mathbf{Z}},$$

where $\rho(dx) = x^{-1}I_{(0,1]}(x)dx$ and η is a σ-finite measure on $\mathbf{R}^{\mathbf{Z}}$ satisfying certain integrability conditions. Another example is the Thorin class of gamma mixtures, where the root Lévy measure is $\rho(dx) = x^{-1}e^{-x}I_{(0,\infty)}(x)dx$.

If $\mathbf{X}$ is a strictly stationary infinitely divisible process without Gaussian part, then η is a σ-finite measure invariant under the shift T. The measure η is assumed to be known (in some form); it plays the role of a spectral measure of the process. In this talk we relate properties of two dynamical systems: probabilistic $(\mathbf{R}^{\mathbf{Z}}, P_{\mathbf{X}}; T)$ and 'deterministic' but possibly infinite system $(\mathbf{R}^{\mathbf{Z}}, \tau; T)$ (here $P_{\mathbf{X}} = \mathcal{L}(\mathbf{X})$). We investigate $(\mathbf{R}^{\mathbf{Z}}, P_{\mathbf{X}}; T)$ as a factor of a lifted probabilistic dynamical system $(\mathcal{M}(\mathbf{R}^{\mathbf{Z}}), P_\eta; T^*)$, which has richer structure and is directly related to $(\mathbf{R}^{\mathbf{Z}}, \eta; T)$. In addition, $L^2(\mathcal{M}(\mathbf{R}^{\mathbf{Z}}), P_\eta)$ admits chaotic decomposition

$$L^2(\mathcal{M}(\mathbf{R}^{\mathbf{Z}}), P_\eta) = \bigoplus_{n=0}^{\infty} \bigoplus_{i_1,\ldots,i_n\in\mathbf{N}} \mathcal{H}^{(i_1,\ldots,i_n)},$$

where $\mathcal{H}^{(i_1,\ldots,i_n)}$ are spaces of multiple stochastic integrals with respect to strongly orthogonal Teugels martingales. The resulting ergodic decompositions of processes into parts with different long range memory structures apply not only to the underlying infinitely divisible processes but also to their nonlinear functionals such as multiple integrals.