The Rate Function of the Large Deviation for a class of Nonhomogeneous Markov Chains

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Abstract: In this talk, we shall consider the LDP for the additive sum $\frac{1}{b_n}\sum_{k=1}^{n}\gamma_k f(X_k)$, where $\{\gamma_k f(X_k), k \geq 0\}$ is a nonhomogeneous Markov chain, $f(.)$ a bounded function and two positive number series $\{b_k, k \geq 1\}$ and $\{\gamma_k, k \geq 1\}$ satisfy $b_k \sim k^\alpha, \gamma_k \sim \gamma\alpha k^{\alpha-1}$ as $k \to \infty$, where $\gamma > 0, \alpha \geq 1$. We show that the rate function $I(y)$ of the LDP has the following expression

$$I(y) = \sup_{\delta>0}\Big\{y\gamma\delta - \int_0^1 \log\rho(\delta\gamma\alpha x^{\alpha-1})\mathrm{d}x\Big\}$$

where $\rho(x)$ is the Perron-Frobenius eigenvalue of the matrix $P(x) = (p_{ij}e^{xf(j)})$, x is a real number and (p_{ij}) is the transition probability matrix of Markov chain $\{X_k, k \geq 0\}$.

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