

An invariance property for the empirical distributions of occupancy problems with application to finance

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Abstract: We prove that the empirical mass functions of a classical occupancy problem converge to that of a geometric distribution a.s. This result is applicable to the case when the return follows a Levy process. The fractional Brownian motion with index in $(0, 1)$ has very different invariance distributions empirically. Note that the empirical invariance found in the market is quite far away from the geometric distribution as well as the empirical distributions obtained from the fractional Brownian motion.

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