ON SWITCHING DIFFUSIONS AND JUMP-DIFFUSIONS

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Abstract: Let $(X(t), Z(t))$ be a right continuously strong Markov process with the phase space $R^d \times N$, where $N := \{1, 2, \cdots, n_0\}$. The first component $X(t)$ satisfies the following stochastic differential equation

$$dX(t) = b(X(t), Z(t))dt + \sigma(X(t), Z(t))dB(t).$$

The second component $Z(t)$ is a discrete random process with the finite state space N such that:

$$P\{Z(t+\Delta) = l | Z(t) = k, X(t) = x\} = \begin{cases} q_{kl}(x)\Delta + o(\Delta), & \text{if } k \neq l, \\ 1 + q_{kk}(x)\Delta + o(\Delta), & \text{if } k = l \end{cases}$$

uniformly in R^d, provided $\Delta \downarrow 0$, where $0 < q_{kl}(x) < +\infty$ for all $k \neq l \in N$. Generally, the strong Markov process $(X(t), Z(t))$ can be called a diffusion process with state-dependent switching. In particular, when the functions $q_{kl}(x)$ are independent of x (i.e., $q_{kl}(x) \equiv q_{kl} > 0$ for all $k \neq l$) and the second component $Z(t)$, which is independent of $B(t)$, is a Markov chain itself, the corresponding strong Markov process $(X(t), Z(t))$ then can be called a diffusion process with Markovian switching. A diffusion process with state-dependent switching or Markovian switching is often called a switching diffusion. The Markov process $(X(t), Z(t))$ is said to be stable in f-norm if there exists a probability measure $\pi(\cdot)$ such that its transition probability $P(t, (x, k), \cdot)$ converges to $\pi(\cdot)$ in f-norm as $t \to 0$ for every $(x, k) \in R^d \times N$. The so-called f-norm is a very strong norm and the well known total variation norm is only a special case of it. Our main aim in the present work is to investigate the stability in f-norm for diffusion processes with state-dependent switching. Actually, we will prove the f-exponential ergodicity for these processes under some reasonable conditions. In the course of pursuing the above objective, we will also prove the Feller continuity for these processes by making use of the Radon-Nikodym derivatives and more the strong Feller continuity for them under some reasonable conditions. Finally, we also give a very brief discussion about the Feller continuity, strong Feller continuity and f-exponential ergodicity for a class of jump-diffusions with Markovian switching.