Essential spectral radius for positive operators on L^1 and L^∞ spaces

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Abstract: For a positive operator π on L^1 and L^∞ spaces over Polish probability spaces, we have proved that

Theorem 0.1 *For an nonnegative operator $\pi : L^p \to L^p$ with $p = 1, \infty$ (if $p = \infty$, π is also a kernel operator) we have*

$$r_{ess}(\pi|_{L^1}) = r_{tail(L^1)}(\pi),\ r_{ess}(\pi|_{L^\infty}) = r_{tail(L^\infty)}(\pi). \tag{1}$$

Proposition 0.2 *Let π be a symmetric positive operator on L^2 with a bounded kernel. Then for any $1 < p < \infty$*

$$r_{ess}(\pi|_{L^p}) \le r_{tail(L^1)}(\pi|_{L^1}) = r_{tail(L^\infty)}(\pi|_{L^\infty}). \tag{2}$$

Proposition 0.3 *Suppose that $Supp(\mu) = E$. Then for a positive Feller kernel operator π on L^∞ we have*

$$r_{tail(L^\infty)}(\pi) = r_{ess}(\pi|_{C_b(E)}). \tag{3}$$

Corollary 0.4 *Suppose that $Supp(\mu) = E$, and the positive Feller operator π satisfying that*

- $r_{tail(L^\infty)}(\pi) < r_{sp}(\pi)$ *(i.e., TNC),*
- π *is topologically transitive,*

where the topologically transitivity of π means that for any $x \in E$ and any nonempty open subset $O \subset E$ there is an integer $N \ge 1$ satisfying that $\pi^N(x, O) > 0$ for the Feller kernel $\pi(x, dy)$ of π. Then π is ergodic in $C_b(E)$ and L^∞.

Proposition 0.5 *For any nonnegative bounded kernel π on $(E, \mathcal{B})$ we have*

$$\sup_{\mu:\ \mu\pi \ll \mu} r_{ess}(\pi|_{L^\infty}) = \sup_{\mu:\ \mu\pi \ll \mu} r_{tail(L^\infty(\mu))}(\pi) \le r_\tau(\pi|_{b\mathcal{B}}) \le r_\Delta(\pi|_{b\mathcal{B}}) = r_{ess}(\pi|_{b\mathcal{B}}). \tag{4}$$

Corollary 0.6 *For any nonnegative bounded Feller kernel π*

$$r_{ess}(\pi|_{C_b(E)}) = \sup_{\mu:\ \mu\pi \ll \mu,\ Supp(\mu)=E} r_{ess}(\pi|_{L^\infty(\mu)}) = \sup_{\mu:\ \mu\pi \ll \mu} r_{tail(L^\infty(\mu))}(\pi) = r_\tau(\pi|_{b\mathcal{B}}) = r_\Delta(\pi|_{b\mathcal{B}}) = r_{ess}(\pi|_{b\mathcal{B}}). \tag{5}$$

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