

# THE LÉVY-FOKKER-PLANCK EQUATION: PHI-ENTROPIES AND CONVERGENCE TO EQUILIBRIUM

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**Abstract:** The paper is written in collaboration with C. Imbert from Paris-Dauphine University (France). We study a Fokker-Planck equation of the form

$$u_t = \mathcal{I}[u] + \operatorname{div}(xu)$$

where the operator  $\mathcal{I}$ , which is usually the Laplacian, is replaced here with a general Lévy operator. We prove by the entropy production method the exponential decay in time of the solution to the only steady state of the associated stationary equation.

Results of the article generalize the paper of BILER and KARCH in [1]. The main tool is logarithmic Sobolev inequality for Lévy process proved by WU in [3] and generalized by CHAFAI in [2].

## References

- [1] P. BILER and G. KARCH (2003), Generalized Fokker-Planck equations and convergence to their equilibria. In *Evolution equations*, volume 60 of *Banach Center Publ.*, pages 307–318. Polish Acad. Sci., Warsaw.
- [2] D. CHAFAI, (2004) Entropies, convexity, and functional inequalities: on  $\Phi$ -entropies and  $\Phi$ -Sobolev inequalities. *J. Math. Kyoto Univ.*, 44(2):325–363.
- [3] L. WU, (2000) A new modified logarithmic Sobolev inequality for Poisson point processes and several applications. *Probab. Theory Related Fields*, 118(3):427–438.