RECENT PROGRESS ON BOUNDARY THEORY OF MARKOV PROCESSES

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Abstract: Given a Markov process X on a state space E and a subset F of E, we may associate the minimal process X^0 on $E_0 = E \setminus F$ and the time changed process Y on F: X^0 and Y are obtained from X by killing upon leaving E_0, and with the time substitution by the inverse of the local time on F, respecively. There are yet another associated process that has attracted the interest of researchers for many years: the excursions of X away from the set F. The boundary theory of Markov processes addresses interrelationship among those objects and concerns how X is determined by X^0 and quantities intrinsic to X^0, and furthermore, based on this information, how extensions of X^0 to E or to some other extended spaces of E_0 can be constructed. I shall talk about some recent progress [6] $\sim$ [15] on this subject with an overview of the historical developments leading to those papers. A key word is *Feller measures* going back to [1] where W. Feller proposed a boundary problem for Markov chains.

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