Collection of Book Reviews

Mu-Fa Chen (editor)

Editor's Note The collection consists of eleven reviews about two books, to be specified in the next page. It devotes to acknowledge the strongly scientific support of the reviewers and to remember their friendship. The reviews are somehow unusual, there are overview at higher level, concrete judgements on the topics studied in the books, and some additional background of the study is also included. A large part of the reviews came from top specialists in the fields. Perhaps, some reader has learnt a part of their names. Honestly, in the past decades, the author has learnt a great deal from the most of the reviewers. Thus, the comments from them, either positive or negative, should be very much helpful not only for the readers but also for the author, and hence are greatly appreciated.

Contents

Review of English abstract of the book "Jump Processes and Interacting Particle Systems" by Mu-Fa Chen

R. L. Dobrushin

October 24, 1988

Jump Markov processes, considered in the book, are one of more well known classes of Markov processes having very numerous applications. In the book several directions of the theory of such processes are studied. There are a lot of interesting results in these directions which were published in recent years by many authors including Chinese ones. Nevertheless these results had not found previously their review exposition in book forms. Some of them were published in Chinese only. The question studied in the book are: uniquness of the construction of process by its infinitesimal characterics, the conditions of differentiability of transition functions and of satisfying Kolmogorov differential equations, method of couplings applicable to the studying ergodic properties of the process and so on. All these results are concretized for an important subclass of symmetrized (reversible) process. Besides it the author applies his result to a very fashionable new class of Markov processes: Markov interacting processes. Here he considers the class of processes with noncompact states including important Schlögl model taken from statistical physics. So the book is an essential addition to the existing literatures on Markov processes. The book is well organized. I can not read the text of the book written now only in Chinese. But I know many English language papers of the author containing important mathematical results and written in a good mathematical style. So I am sure about the quality of the exposition. I can surely recommend the book for the publication in English translation. I have a private interest to such project because I want to have a possibility to read the book.

1993 年当选美国科学院外籍院士

R. Dobrushin

MR1168209 (94a:60135) 60K35 (60J27 82B20 82C22)

Chen, Mu Fa (PRC-BJN)

★**From Markov chains to nonequilibrium particle systems.**

World Scientific Publishing Co., Inc., River Edge, NJ, 1992. x+550 *pp*. $58.00.
ISBN 981-02-0639-9

Two areas of probability theory to which Chinese probabilists have made substantial contributions over the years are continuous-time Markov chains and interacting particle systems. A major accomplishment in the first area was Z. T. Hou's solution in 1974 of a uniqueness problem for Q-processes. More recently, the school led by the author in Beijing has worked on the construction and ergodic theory of the class of interacting particle systems known as reaction diffusion processes. This book is an account of progress in these two areas, concentrating on the Chinese contributions. It is based on an earlier book by the author [*Jump processes and particle systems* (Chinese), Beijing Shifan Daxue Chubanshe, Beijing, 1986; MR0882543 (88e:60100)], but contains a large amount of new material.

Following an introductory chapter, the book consists of four parts, titled, respectively, General jump processes, Symmetrizable jump processes, Equilibrium particle systems and Nonequilibrium particle systems. The first part deals with issues of existence and uniqueness of Q-processes, and their classification according to recurrence properties. The second part is a study of reversible chains, including their large deviation properties and estimates on the spectral gap. Part III discusses the Ising model, reversibility of spin systems and exclusion processes, and Yang-Mills lattice fields. The final part consists of the construction of interacting particle systems, with emphasis on systems with noncompact state spaces, criteria for ergodicity, and hydrodynamic results for reaction diffusion processes. The treatment of interacting particle systems is not complete, and is not intended to be. For example, the contact process is not mentioned, and there is only brief mention of the exclusion process and the stochastic Ising model. There are sections at the end of each chapter which provide references and further information. There is also an extensive bibliography with over 200 entries.

This book provides a useful account of a substantial portion of the work of Chinese probabilists over the past two decades, much of which has been relatively inaccessible to Western workers. The English is somewhat rough, but not so much as to interfere with communication. There are some other imperfections. For example, on page 1 the derivatives of the transition probabilities of a Markov chain at $t = 0$ are written awkwardly as $(1 - \delta_{ij})q_{ij} + q_{ii}\delta_{ij}$ instead of q_{ij}. On page 5, the symbol $P^{\min}$ occurs, and on page 13 there is a reference to the "minimal Q-process", but as far as the reviewer could determine, the important concept of minimal process is not introduced until page 70. Still, this is a useful reference work for probabilists working in these areas, and a contribution to international communication in probability theory.

2008 年当选美国科学院院士

Reviewed by *T. M. Liggett*

SIAM Review, Vol. 35, No. 3.
(Sep., 1993), p. 514.

5

From Markov Chains to Nonequilibrium Particle Systems. By *Mu Fa Chen*. World Scientific, Singapore, 1992. x + 550 pp. $ 58.00, hardback. ISBN 981-02-0639.

It is remarkable that this book was ever written. Chen Mu Fa's research was interrupted early in his career by the cultural revolution. When that was over he got back to work learning interacting particle systems, which was then an emerging branch of probability theory. He did a lot to popularize the subject in China and with Yan Shi-jian was instrumental in having the second special year 1988-89 at the Nankai Institute devoted to probability. At the end of that year the events at Tian An Men square occurred, creating turmoil in Beijing. But Chen and his group evidently found a way to keep working since the book was completed in November 1991.

The book is admirable not only for the circumstances in which it is written but for what it contains. It begins with a treatment of continuous time Markov chains on a countable state space to set the stage for the consideration of particle systems which are continuous time Markov chains with state space $\{0,1\}^S$ and $\{0,1,2,...\}^S$, where S is a countable set, usually the d-dimensional integer lattice. The second part of the book considers reversible Markov chains and their connections with electrical networks, then considers large deviations problems and conditions for the existence of a spectral gap.

Part three concerns the Ising model and related models from statistical physics. In addition to standard topics such as Peierl's argument for the existence of phase transitions and the hard to find results on the form of reversible spin systems, there are more modern ideas such as reflection positivity and chess board estimates, and some new research on Yang-Mills' lattice fields and the Ising model on lattice fractals. Part four concerns the time evolution of interacting particle systems concentrating on what the authors call reaction diffusion processes, in which particles perform independent random walks on the lattice and at each site particles are born or die at a rate that depends upon the number of particles at that site. Chen presents results on the construction of the process, and sufficient conditions for ergodicity and phase transitions some of which are published here for the first time, then closes the book with a consideration of "hydrodynamic limits" which arise when the particles move fast and the lattice is scaled.

As the quick summary above should indicate, Chen's book contains a wide variety of topics that cannot be found in any other place. It is not as carefully and clearly written as Liggett's Interacting Particle Systems, but the occasionally unusual English does not really get in the way. The bottom line is that if you are curious about interacting particle systems this book belongs on your bookshelf. If not, you might consider buying this book as a political statement in support of Chinese probabilists who continue to do hard work in difficult circumstances or in support of a publisher who brings out nice looking books at about 10.5 cents a page.

RICK DURRETT
Cornell University

0753.60055

Chen, Mu Fa

From Markov chains to non-equilibrium particle systems. (English)
Singapore: World Scientific. x, 550 p. (1992). [ISBN 981-02-0639-9]

The book is a comprehensive account of the theory of jump processes and particle systems. The author is an outstanding Chinese specialist in probability theory and stochastic processes creating the Chinese school of Markov processes. The material is divided into five parts: (1) Overview and preliminary results: classical Markov chains, coupling ideas, large deviations, particle systems. (2) General jump processes: transition function and Laplace transform, existence and construction of jump processes, uniqueness criteria, recurrence, ergodicity and invariant measures, probability metrics and coupling methods. (3) Symmetrizable jump processes: symmetrizable jump processes and Dirichlet forms, field theory, large deviations, spectral gap. (4) Equilibrium particle systems: random fields, reversible spin processes and exclusion processes, Yang-Mills lattice field. (5) Non-equilibrium particle systems: construction of the processes, existence of the stationary distributions and ergodicity, phase transitions, hydrodynamical limits.
Reviewer's remark: Probability metrics and related coupling ideas are thoroughly studied in the books of the reviewer [Probability metrics and the stability of stochastic models (1991; Zbl 0744.60004)] and of *V. V. Kalashnikov* and the reviewer [Mathematical methods for construction of queueing models (1990; Zbl 0709.60096)].

S.Rachev (Santa Barbara)
Keywords : theory of jump processes; particle systems; Markov chains; coupling ideas; large deviations; spectral gap; Equilibrium particle systems; Yang-Mills lattice field; hydrodynamical limits
Classification :
∗60Jxx Markov processes
60-02 Research monographs (probability theory)
60Axx Foundations of probability theory
60Fxx Limit theorems (probability)
81T13 Gauge theories
60Exx Distribution theory in probability theory
82B20 Lattice systems
82C22 Interacting particle systems
Cited in ...

陈木法教授所著《From Markov Chains to Non-equilibrium Particle Systems》是一部高水平的学术专著. 其目的有二: 一是论述 Markov 过程理论的最新进展; 二是将理论运用于研究物理中的无穷粒子系统. 分述如下:

(一) 关于 Markov 过程的理论. 作者深入而且尽可能全部地研究了跳跃型过程(简称跳过程), 其中总结了作者本人及国内同行的研究成果, 并有新的发展. 同时为完整起见, 也论述了国外的一些新成果. 关于一般空间中的跳过程的专著, 就我所知, 本书在国际上是第一本. 本书内容又极新颖, 其中包括过程的存在性, 唯一性, 过程的构造, 常返性, 不变测度, 对称跳过程与狄氏型, 谱隙论等. 特别是作者(与侯振挺合作) 首先引进的场论及其在 Markov 链中的应用, 是一创举. 作者的另一重要贡献是发展了耦合(Coupling)理论, 并用此工具取得许多优美的新成果, 例如用以研究 Riemann 流型上拉氏算子的第一特征值等.

(二) 关于对无穷粒子系统的应用. 作者首先论述平衡态粒子系统. 作者运用自己所首创的场论方法, 得到了许多新结果. 例如粒子系统可逆性的简捷判别法, 这项成果后来为许多人所引用. 继平衡系统之后, 本书达到高潮, 即研究非平衡态粒子系统. 陈木法在国际上最早引进无穷维反应扩散过程, 并以此作为非平衡统计物理的典型模型, 对它进行了深入的研究, 得到了许多新结果. 其中包括作者的构造定理, 此定理被国外学者誉为"陈氏构造定理".

本书出版后获得国际上许多著名概率学家如 S. Rachev (俄), T.M. Liggett (美), R. Durrett (美)等人的高度评价. 例如 R.Rachev 写道: "本书是对跳过程与粒子系统理论的全面总结. 作者是概率论和随机过程方面杰出的中国专家, 创建了 Markov 过程的中国学派....." (Zbl. Math. v.753, p.301).

本书既有很高的理论水平, 又深入物理实际, 因而是理论联系实际的大部头杰作(全书共 551 页). 其中许多精彩篇章如关于跳过程的最新成果, 关于场论方法, 关于首创的反应扩散过程, 关于耦合理论的新发展和巧妙运用, 构成本书的鲜明特色. 本书有很高的学术价值, 无疑地具有国际领先水平. 因此, 我谨竭诚地推荐本书为霍英东教育基金会青年教师基金项目的优秀成果.

北京师范大学数学系　王梓坤教授(中科院院士)　1996.1.20.

1991 年当选中国科学院院士

MR2091955 (Review)

Chen, Mu-Fa(PRC-BJN)

From Markov chains to non-equilibrium particle systems.

Second edition.
World Scientific Publishing Co., Inc., River Edge, NJ, 2004. xii+597 pp. *ISBN* 981-238-811-7
60-02 (37A30 47D07 60J27 60J75 60K35 82C22)

Among all continuous time Markov process classes, Markov chains and, more generally, jump processes are certainly the easiest to introduce. Nevertheless, given a Q-matrix (or a transition rate kernel), three fundamental problems already appear: the uniqueness, the recurrence and the ergodicity of corresponding semigroup(s). So the first part of this book studies these questions in welcome detail. The tools are analytical (resolvents, Kolmogorov backward and forward equations, minimal semigroups, entrance and exit spaces, etc.) or probabilistic (behavior of return times, couplings, etc.) and often a mix of both. The second part deals with the corresponding L^2-approach for symmetrizable jump processes. This leads the reader from basic Dirichlet form theory to entropy rate functions for large deviations of Markovian occupation times or to spectral gap estimates. The third part gives a survey of some results about equilibrium particle systems, especially Ising models (on Euclidean lattices or fractals), spin and exclusion stochastic models and Yang-Mills lattice fields. The corresponding reversible processes are no longer jump processes, but can be conveniently approximated by such processes (extensions to diffusion processes are also considered) through restrictions to finite boxes of sites. In contrast, the last part is concerned with non-equilibrium particle systems, i.e., systems whose associated processes are in general non-reversible and thus even harder to handle. The typical example is Schlögl's model (as the author says, the topics studied in almost every chapter of his book are either led by or related to this model) and its generalizations: polynomial models and reaction-diffusion processes. As before, first the processes are constructed and their stationary distributions and ergodicity properties are studied. Next, phase transitions and hydrodynamical limits are investigated for some concrete models.

This book is the second edition of \ref[M. F. Chen, *From Markov chains to nonequilibrium particle systems*, World Sci. Publishing, River Edge, NJ, 1992; MR1168209 (94a:60135)] and contains numerous modifications and corrections. The two chapters concerned with probability metrics, coupling

methods and with spectral gaps have been rewritten because these fields have been very active recently, in particular due to the contributions of the Beijing school led by Mr. Chen.

Sometimes the notation is disturbing, for instance, $P(t)$ designates either a whole semigroup or its particular operator at time t and furthermore the same symbol, $P(\lambda)$, is used for its Laplace transforms. Nevertheless, this book should be very useful to anyone interested in Markov processes, because it can serve as an introduction to a large variety of subjects and models, as well as an account of some recent works of Chinese probabilists (in this respect, another book more focused on ergodic inequalities is announced by Mr. Chen).

Reviewed by *Laurent Miclo*

American Mathematical Society
201 Charles Street
Providence, RI 02904-6248 USA

Zbl 1078.60003

Chen, Mu-Fa

From Markov chains to non-equilibrium particle systems. 2nd ed. (English)

River Edge, NJ: World Scientific. xii, 597 p. £ 63.00; $ 104.00/hbk; $ 252.00/ebook (2004). ISBN 981-238-811-7/hbk; ISBN 978-981-256-245-6/ebook
http://ebooks.worldscinet.com/ISBN/9789812562456/toc.shtml

For the first edition (1992) see Zbl 0753.60055. As the author mentions, the main changes concern the chapters on probability metrics and spectral gap. Besides, the references have been updated and numerous corrections and small modifications of presentation undertaken. Some sections are based on new results by the author and his colleagues.

The fifteen chapters of this book are split into four parts. Each chapter ends up with (a bit too short) sections with references and historical notes. In the first part the author discusses Markov processes on a general state space. The main tool are the rates (or derivatives) of the transition probabilities called q-functions or Q-matrices. Using analytical tools (transition kernels, resolvents, Kolmogorov equations) the author addresses the issues of the uniqueness and existence problem for the processes with given q-functions. A particular attention is devoted to jump processes. Then the author discusses the issues of recurrence, ergodicity, invariant measures, embedded jump process, and birth-death processes. Finally, this part provides a good exposition of the Kantorovich metric and related coupling issues. It should be noted that the author seldom uses the tools based on the infinitesimal generator and martingale methods.

Part II begins with the study of reversible and symmetrizable jump processes and Dirichlet forms. Then the author describes main concepts of the field theory (that describes the communicating states and the potential differences corresponding to the allowed transitions), in particular, electric fields and their application to the study of transience for symmetrizable Markov chains and random walks on fractals. Starting from the first principles, the author elaborates the large deviation techniques for Markov chains. This part ends up with the discussion of spectral theory, Dirichlet forms and isoperimetric (Cheeger's) constants.

Part III concerns equilibrium particle systems, in particular Gibbs fields on the grid, related existence and uniqueness issues, and the Peierls method. Further topics include the reversible spin process and exclusion process, Yang-Mills lattice field and the related diffusion process. Part IV deals with non-equilibrium particle systems, construction and existence, their stationary distributions, phase transitions and hydrodynamic limits. The Schlögl model of chemical reaction with diffusion is used to illustrate several results throughout the book, especially those concerning non-equilibrium systems.

The index is a bit too short for the book of this length. For instance, the index does not include the q-process, which is one of the principal objects of this book. It is a pity that a list of notations is not present, since many results are formulated using only mathematical symbols without recalling in words their meaning. However, the book is a welcome addition to the Markov processes literature and will be useful to anyone

working in this area, especially for specialists in interacting particle systems, Gibbs processes and probabilistic aspects of statistical physics.

Ilya S. Molchanov (Bern)

Keywords : Markov process; Gibbs field; spin process; reaction-diffusion process; jump process

Classification :

∗60-02 Research monographs (probability theory)
60J27 Markov chains with continuous parameter
60K35 Interacting random processes
82B20 Lattice systems

12 From Markov Chains to Nonequilibrium Particle Systems (second edition) By Mu-Fa Chen.

During the past years the areas of mathematical physics turn out to be more and more connected with the probabilistic construction, in particular with the stochastic process. The Mu-Fa Chen's monograph connects modern theory of random processes and the mathematical physics. In that book very complete explanation for some class of stochastic processes – so called jump Markov processes – is given together with large spectrum of the problems from the domain of the mathematical physics. I give short description of content of this book.

The first part is devoted to the foundations of the Markov process theory and in particular to the jump processes. Here some classical questions from that theory are discussed: the existence of the processes with given infinitesimal transition probabilities; uniqueness of such process, its properties – recurrence, ergodicity, mixing and so on. Simultaneously some technical tools are introduced, for instance, the coupling methods (it should be said that the author of the book used this method in his works very virtuously). Besides general theoretical explanation, very many concrete examples and problems are considered.

In the second part, which continues in the same spirit as first one, the theory of the symmetric (reversible) processes is explained. Here the same questions as before – existence, uniqueness and so on – are studied. Further L_2-theory of such processes is introduced including the theory of Dirichlet forms. In the last chapter of this part one can find some mathematical applications of the general theory: theory of the fields, large deviations, and spectral gap in the spectrum of the generator of Markov process.

In the third part of the book the author comes to the problems from the domain of mathematical physics: the basis notions and the tools of so-called DLP-theory, phase transitions for lattice systems and the most known methods in this theory – Peierls (contour) method, reflection positivity and, connected with them, so-called Chess-board estimates. Two last chapters of this part are devoted to stochastic dynamics for spin systems (Glauber dynamics and excusion process) and also lattice processes connected with Yang-Mills fields.

Finally the last (the fourth) part of the book contains some fragments of the theory of non-equilibrium systems: the constructions of several typical models of such processes, the questions of egrodicity and existence of stationary state for them, the appearance of phase transitions and hydrodynamical behaviour for these systems.

From this short review of book one can see how much diverse and rich material is contained in it. The book is useful for probabilists who are interested in physical application of random processes and, of course, to mathematical physicists, who can use it as a guide for random processes.

The second edition almost does not differ from the first one by content, but two chapters are rewritten, some materials are updated, and its exterior became essentially better.

R. Minlos
8 April 2005

SIAM REVIEW
©2006 Society for Industrial and Applied Mathematics
Vol.48, No.1 , pp. 149–204

13

Book Reviews Edited by Robert E. O'Malley, Jr .

Eigenvalues, Inequalities and Ergodic Theory. By Mu Fa Chen.
Springer-Verlag, London, 2005.
$89.95. xiv+228 pp., hardcover. ISBN 1-8523-3868-7.

This book is like a Chinese buffet restaurant. You won't find lasagna or roast beef, but there is a wide variety of good food to eat.

The main topics of this book, as indicated in the title, are eigenvalues, inequalities, and ergodic theory. Specifically if one has a Markov chain or diffusion process, there is an associated difference or differential operator. If the operator is self-adjoint, then the eigenvalues are nonpositive real numbers starting with $\lambda_0 = 0$. The gap to the next one, $\lambda_1 < 0$, can be used to bound the rate of convergence to equilibrium of the associated Markov process, but for differential geometers and their Laplacians, it is an interesting object in its own right.

Inequalities for the spectral gap and their applications are a huge topic. Chen's 228-page book takes the reader on a tour of some of this material. The emphasis is on topics that Chen has contributed to (and I think this is appropriate), but this includes a wide variety of interesting topics. After a one-chapter overview, the book begins with optimal couplings, new variational inequalities, and Cheeger's method. The focus then shifts to one-dimensional processes for which a very detailed theory is possible. Chapter 8 is devoted to a proof of Figure 1.1, which relates nine different notions of ergodicity. The final two chapters consider interacting particle systems of the reaction-diffusion type and some problems that arise in economics.

The author has lived an interesting life. His first publication listed in the book is from 1984. Since then individual freedom in China has expanded and contracted, but Chen has carried out an active research program and traveled extensively (as indicated by the long list in the acknowledgments). Chen is an experienced researcher who has won prizes in China and spoken at the International Congress of Math, and he is an extensive writer of lecture notes. One might ask for the book to cover a broader range of topics, but one cannot complain about how it is written. The math is clearly explained and the book is pleasant to read. This book won't solve all of your problems in this important subject, but it deserves a place on your bookshelf.

RICK DURRETT
Cornell University

MR2105651 (2005m:60001) 60-02 (26D15 34L15 35P15 58J50 60J10 60K35)

Chen, Mu-Fa (PRC-BJN)

★**Eigenvalues, inequalities, and ergodic theory.**

Probability and its Applications (New York).

Springer-Verlag London, Ltd., London, 2005. *xiv*+228 *pp*. $89.95. *ISBN* 1-85233-868-7

This book is mostly based on the author's results on various problems related to inequalities arising in the theory of Markov chains with emphasis on the problem of estimating the first nontrivial eigenvalue of the Markov operator and its relation to the speed of convergence of the Markov chain to the stationary distribution. The book consists of 10 chapters: An overview of the book (1), Optimal Markov couplings (2), New variational formulas for the first eigenvalue (3), Generalized Cheeger's method (4), Ten explicit criteria in dimension one (5), Poincaré type inequalities in dimension one (6), Functional inequalities (7), A diagram of nine types of ergodicity (8), Reaction-diffusion processes (9), Stochastic models of economic optimization (10) and two appendices: Some elementary lemmas (A) and Examples of the Ising model on two or four sites (B).

Reviewed by *Vadim A. Kaĭmanovich*

1079.60005

Chen, Mu-Fa

Eigenvalues, inequalities, and ergodic theory. (English)

Probability and Its Applications. London: Springer. xiii, 228 p. EUR 89.95/net; sFr 152.50; £50.00; $ 89.95 (2005). [ISBN 1-85233-868-7/hbk]

In short, a problem of broad interest – the estimation of the spectral gap for matrices or differential operators (Markov chain or diffusions) – is covered in this book, and the book surveys, in popular way, the main progress made in the field. The book consists of 10 chapters, two appendices and more than 300 references. The first chapter is an outline of the second to the eight chapters. Several different inequalities and different types of convergence by using three mathematical tools, a probabilistic, the coupling methods and a generalized Cheeder's method, are presented in Chapters 2, 3 and 4, respectively. The explicit criteria for different types of convergence and the explicit estimates of the convergence rates in dimension one are given in Chapters 5 and 6. Some generalizations of functional inequalities are considered in Chapter 7. Chapter 8 contains the proofs of a diagram of nine types of ergodicity. Chapter 9 surveys the study of a class of interacting particle systems and illustrates some applications. The last Chapter 10 contains an application of the first eigenvalue, its eigenfunctions, and an ergodic theorem to stochastic models of economics. Appendix A surveys some elementary lemmas and Appendix B gives some examples of the Ising model on two to four sites. Each chapter starts with a summary, and ideas are introduced through simple examples rather than technical proofs, which is one of the advantages of the book, in order to appeal to nonspecialists. This book may be useful for researchers, graduates and postgraduates in probability theory, Markov processes, mathematical physics and spectrum theory.

Anatoliy Swishchuk (Calgary)

Keywords : Newman and Dirichlet eigenvalues; Poincaré, Sobolev, Nash inequalities; ergodic processes; variational formula; generalized Cheeder's method; Markovian coupling; reaction-diffusion processes; economic optimization

Classification :

∗60-02 Research monographs (probability theory)
60J25 Markov processes with continuous parameter
60K35 Interacting random processes
37A25 Ergodicity, mixing, rates of mixing
37A30 Ergodic theorems, spectral theory, Markov operators
47A45 Canonical models for contractions and nonselfadjoint operators
58C40 Spectral theory on manifolds
34B24 Sturm-Liouville theory
34L15 Estimation of eigenvalues for OD operators
35P15 Estimation of eigenvalues for PD operators
91B02 Fundamental topics on applicability to economics

Cited in ...