

## Summary of Growth of Mathematical Stochastics

Mu-Fa Chen

(1. Research Institute of Mathematical Science, Jiangsu Normal University, Xuzhou, Jiangsu, 221116, P. R. China; 2. School of Mathematical Sciences, Key Laboratory of Mathematics and Complex Systems (Ministry of Education), Beijing Normal University, Beijing, 100875, P. R. China)

**Abstract:** This paper summarizes the growth and grow up of mathematical stochastics. It includes two periods of early history, the syllabus of “College Mathematics” for non-mathematics majors facing the new century, the teaching materials and research institutions of “mathematical stochastics”, the mark of grow up of mathematical stochastics, etc.

**Keywords:** Probability theory; mathematical statistics; mathematical stochastics; textbook; prize.

**MSC (2020) Classification:** 60-XX; 62-XX.

### 1 Two brief histories

#### Early period

Mathematical Stochastics became officially a branch of mathematics dates back to 1933. In that year, the Russian mathematician A.N. Kolmogorov established the first axiomatic system of probability theory<sup>[16]</sup>. However, as the American mathematician W. Feller wrote in the preface to his two volumes [12, 13]: during the period of his writing (1941–1948), except the former Soviet Union, only a few of mathematicians admitted probability as a legitimate branch of mathematics. The development of mathematical stochastics in China is due to the foresight of the elder mathematicians in our country. When they formulated the outline of the long-term plan for the development of science and technology in the country from 1956 to 1967, they listed the following three subjects “Computational Mathematics, Probability Theory and Mathematical Statistics, Differential Equation Theory” as “an important, urgently needed and blank or weak part in mathematics that needs energetically development”<sup>[14]</sup>. Along this line, further development can be seen from the book [1] which is devoted in memory of the centenary of Professor Bao-Lu Xu, who was honored by Professor Luo-Geng Hua as the commander-in-chief of China’s probability and statistics. Professor Xu made a lot of foundational contributions to mathematical stochastics in China in the 1950s. In pages 366 and 367 of the book (from Professor Di-He Hu’s memories), three measures were taken to implement the 12-year-long-term plan under the leadership of Professor Xu: (1) Concentrate strength and cultivate talents; (2) Recruit foreign experts to give lectures in China (author’s note: there were already several ones in China at that time); (3) Import textbooks and update teaching contents. Undoubtedly, it was the very important beginning of mathematical stochastics in our country, and brought up the first batch of backbone talents.

## The period of great development

Until the early 1960s, There are only three main branches in probability theory: limit theory, stationary processes and Markov processes (cf. [17; preface]).

The great development of mathematical stochastics began in the mid-1960s. During that period, mathematics began to return to nature, that is, from the Hilbert era of the axiomatic movement back to the Poincaré era. The earliest step was made by R.L. Dobrushin's school of the former Soviet Union where probability theory and (equilibrium) statistical mechanics were crossed, and then F. Spitzer's school of the United States in the 1970s. There are also new branches such as percolation theory and random processes in random environments, etc. By the mid-1970s, new branches such as stochastic analysis, stochastic differential geometry, and large deviation theory have emerged (It is easy to imagine that many stories and references are omitted here. We mention only that in the past years, about every decade, an overview by the author on the progress of the subject was presented: [3, 4, 7, 8], from which one can find additional information). Due to the progress just mentioned, the magazine "Stochastics" was founded in Britain in 1975. As far as we know, this may be the first time at which the term "stochastics" appeared in the literature. In fact, the word was excluded even in 2020 in the books [2, 15]. In order to find out whether more people use this word in today's world, a few days ago, we use Google search with quotation marked "stochastics", the outputs were about 2.3 million web pages. This was quite shocking to the author.



"stochastics" 230 万

176 万

找到约 2,300,000 条结果 (用时 0.71 秒)

### List of issues Stochastics An International Journal of ...

**Stochastics**: An International Journal of Probability and Stochastic Processes (2005 - current).  
Formerly known as. **Stochastics** and Stochastic Reports (1989 - ...

en.wikipedia.org › wiki › Stochastic ▼ 翻译此页

### Stochastic - Wikipedia

Stochastic refers to the property of being well described by a random probability distribution. ... in 1713, Jakob Bernoulli used the phrase "Ars Conjectandi sive Stochastice", which has been translated to "the art of conjecturing or **stochastics**".

[Stochastic oscillator](#) · [Stochastic optimization](#) · [Stochastic matrix](#)

en.wikipedia.org › wiki › Stochastic\_oscillator ▼ 翻译此页

### Stochastic oscillator - Wikipedia

**Stochastics** attempts to predict turning points by comparing the closing price of a security to its

As a comparison, we also inquired about a century-old university in China (using the full English name of the university), the number of web pages is approximately 1.76 million. Hence the term “stochastics” is now quite popular internationally. Enter into the webpages, one can see entries including “Institute of Stochastics” and “Mathematical Stochastics”. By the way: we suggest that the latter one as the English name of the subject (instead of direct translation “Stochastic Mathematics”). This is because on the one hand it matches the traditional English translation of “Mathematical Statistics”; and on the other hand, as a noun, “stochastics” can be used alone, reflecting the essential attribute of “randomness”.

## 2 “College Mathematics” for the new century syllabus and textbooks on “Mathematical Stochastics”

About 24 years ago, commissioned by the Ministry of Education, Professor Shu-Tie Xiao, chairman of the department of applied mathematics at Tsinghua University, formulated a new century syllabus of “College Mathematics” for non-mathematics majors, and organized a special seminar. I had the honor to give a frontier report on probability and statistics at the conference. After 20 years of cross-research on probability theory with statistical physics, analysis and differential geometry in mathematics, the author feels that the title of the course as well as teaching contents of “Probability Theory and Mathematical



Statistics” could be updated. So I boldly suggested that it would be renamed as “Mathematical Stochastics”. The trigger for this name is the “stochastics” mentioned above. Surprisingly, the proposal was adopted immediately. Subsequently, the textbook on “Mathematical Stochastics” edited by Professor Shu-Tie Xiao, with authors Min-Ping Qian at Peking University and Jun Ye at Tsinghua University was published with two editions: 2000, 2004.

The following is the **textbooks on “Mathematical Stochastics”** that the author recently found.

- 1) College Mathematics: [Mathematical Stochastics](#), Shu-Tie Xiao (editor), Min-Ping Qian, Jun Ye (authors), Higher Education Press 2000/2004.
- 2) Handbook of Modern Mathematics: Volume of [Mathematical Stochastics](#), Xi-Ru Chen, Zhong-Guo Zheng (editors), Huazhong University of Science and Technology Press 2000.
- 3) College Mathematics: [Mathematical Stochastics](#), Wen-Sen Gao, Higher Education Press 2004.  
[Mathematical Stochastics](#) (2nd ed.), Zhong-Fan Li, Yi Sun, Wen-Sen Gao, Higher Education Press 2009.  
[Mathematical Stochastics](#) (3rd ed.), Yi Sun, Yan-Wei Gao, Jing Zhang, Higher Education Press 2014.
- 4) Fundamentals of [Mathematical Stochastics](#), Zhen-Hua Cao, Higher Education Press 2009.
- 5) Introduction to [Mathematical Stochastics](#), Yuan-Lie Lin, Zong-Xia Liang, Tsinghua University Press 2003.
- 6) Fundamentals of Financial [Mathematical Stochastics](#), Qi-Kang Ran, China Machine Press 2017.
- 7) [Mathematical Stochastics](#), Yu Feng, Ping Chen, Chuan-Zhi Hou, National Defense Industry Press 2008.
- 8) Introduction to [Mathematical Stochastics](#), Pei-De Chen, Science Press 2001.
- 9) Fundamentals of [Mathematical Stochastics](#), Zheng Tian, Hua-Yong Xiao, etc, Higher Education Press 2005.
- 10) Fundamentals of Engineering [Mathematical Stochastics](#), Xian-Ze Xu, Wuhan University Press 2013.
- 11) [Mathematical Stochastics](#) and Its Applications, Guo-Qing Liu, Xiao-Hui Ai, Harbin Institute of Technology Press 2012/2018.
- 12) [Mathematical Stochastics](#) and Its Applications, Ping Chen, Chuan-Zhi Hou, Yu Feng, People’s Posts and Telecommunications Press 2015.
- 13) [Mathematical Stochastics](#) Modeling Methods and Their Applications, Ming Ma, Science Press 2013.

The first two books were both published in 2000. The former one is in half a year earlier and puts the emphasis on probability; while the latter one puts the emphasis on statistics. Both the cover of the first book and the preface of



the second one indicate that they are works for the 21st century. In fact, the series of the textbooks on “College Mathematics” edited by Professor Shu-Tie Xiao are the basic textbooks designed for non-mathematics majors in Chinese colleges and universities. There are five main subjects: Unary Calculus (2000), Multivariate Calculus (2000), Mathematical Experiments (1999) (It should be mentioned that a working group, in charge of the author, on mathematical software had worked on the construction of this course for five years, see [11]), Algebra and Geometry (2000), Mathematical Stochastics (2000). The “Handbook of Modern Mathematics” consists of five volumes (5000 pages in total), published by Huazhong University of Science and Technology Press and edited by Professor Li-Zhi Xu: Classic Mathematics (2000), Modern Mathematics (2001), Computer Mathematics (2001), Mathematical Stochastics (2000), Economics Mathematics (2001). Interestingly, both sets of books have a common title “Mathematical Stochastics”. There are co-authors in three editions of the third book, so it is regarded as one book here. The first and eleventh books were both published in two editions. Perhaps most of these textbooks are written for undergraduates; but the eighth book is the textbook of the graduate school of the Chinese Academy of Sciences.

There are also some **Mathematical Stochastics Research Institutions in China, and a National Key Specific Project.**

- 1) The Research Center for [Mathematical Stochastics](#) was founded in 2001 at Beijing Normal University with the support of the Innovation Research Team of the National Foundation of China.
- 2) [Mathematical Stochastics](#) Research Center at Nanjing University of Posts and Telecommunications (2018).
- 3) Key Laboratory of “Computation and [Mathematical Stochastics](#)” of Ministry of Education at Hunan Normal University (2018).
- 4) Beijing Normal University renamed the Teaching and Research Section of Probability and Statistics into the Teaching and Research Section of [Mathematical Stochastics](#) (2020).
- 5) The research project “[Mathematical Stochastics](#) and Cross-Research of Mathematics and Physics” at Beijing Normal University in collaboration with Peking University and Beijing Institute of Applied Physics and Computational Mathematics, received the key special funding of the National Key Research and Development Program “Key Scientific Issues of Transformational Technology” and was launched in 2021.

### 3 Signs of grow up of Mathematical Stochastics

#### Awards

- At least one person has won the [Fields Medal](#) in each of four Congress of Mathematicians held since 2006 (none previously). A total of 6.5

persons. For example, there were 4 winners at ICM 2006 and their research fields are as follows:

[Andrei Okounkov](#): probability theory, representation theory, algebraic geometry.

[Wendelin Werner](#): geometric and conformal mapping of two-dimensional Brownian motion.

Grigori Perelman: analysis and geometric structure of Ricci flow.

[The logarithmic Sobolev inequality](#) was used in his proof of the Poincaré conjecture.

Terence Tao: PDE, combination, harmonic analysis and additive number theory, [random matrix](#).

The chairman of the International Mathematical Union (IMU), John Ball pointed out: “[Probability swept most of the award](#)”, “[2.5 of them, to be exact](#)”.

- The first [Gauss Prize](#) in Applied Mathematics was awarded to K. Itô by the International Mathematical Union (2006), see [8].
- The [Abel Prize](#) was awarded [three times](#) to (i) S.R.S. Varadhan (2007) (also see [8]). (ii) Ya.G. Sinai (2014): The award ceremony commends him for “his fundamental contributions in dynamical systems, ergodic theory and mathematical physics”. (iii) H. Furstenberg and G. Margulis (2020): The awards ceremony commended them for “the first to propose methods of using probability theory and dynamical systems in group theory, number theory and combinatorics”. The award has been awarded 18 times in total. By the way, in the popular science works on the author’s homepage, one may find many interactions between our research group and the Dobrushin & Sinai’s team.

## Another sign

As we all know, mathematical stochastics has grown under the nurturing of other branches of mathematics (especially analysis). In recent decades, a new sign of grow up of the subject has appeared. Mathematical stochastics has not only constantly opened up new fields, but also [solved directly the core problems in other fields](#). Here are two examples chosen randomly

- V. Krylov and M.V. Safonov (1979): estimates of completely nonlinear equations.
- H. Furstenberg (1970s): prime numbers exist long arithmetic series.

The latter is one of the achievements of the Abel Prize in 2020. More examples can be found in [8]. Perhaps, from a number of the author’s works [5, 6, 9, 10], one can also find some examples of the interactions of mathematical stochastics and statistical physics, analysis and geometry, quantum mechanics, etc. (most of the articles and related videos listed here can be found from the author’s homepage (at the end of the article)). [Mathematical stochastics and](#)

related interactions have become a fashion and big trend. The idea of randomness has deeply entered into the fields of mathematics and many other natural sciences, and has become an important tool in most scientific fields such as artificial intelligence (especially reinforcement learning) and computational mathematics. In fact, this phenomenon is not surprising. Because of the three major elements of philosophy: unity of opposites, change in quantity and change in quality, chance and necessity, one of them contains mathematical stochastics. It can be seen that **ignoring mathematical stochastics will definitely suffer losses!** As a concise summary of the development of modern mathematical stochastics, here is the first few sentences of the preface written by Professor Bo-Ju Jiang for the commemorative collection [1]: “This year is the 100th anniversary of the birth of Professor Bao-Lu Xu. He is the founder of mathematical statistics and probability theory in China. Now, the world has entered into the information stage, and mathematical statistics has developed into a major aspect of mathematical science which is extremely closely related to science, technology, economy, and society. The idea of probability theory has also powerfully penetrated into every branch of core mathematics, and played a unique and magical role.”

It should be pointed out that this article is inevitably one-sided. As “the half of the sky” of mathematical stochastics, the mathematical statistics is hardly involved; even for probability theory, only a few points for which the author is more or less familiar are mentioned; moreover, the paper lacks analysis and prospects for the future (with a slight exception in article [9]). The author expects that the works of experts will give us a more comprehensive understanding of mathematical stochastics and add new impetus for the future development.

At this point, it may give people an extreme impression. In fact, the author advocates that mathematicians should learn a little ecology, the core of which is “**species coexistence**”. What would the world be like if everyone come to raise tigers, since the tigers were the most powerful species in the world? I remember that Australia used to pass laws to kill a group of kangaroos because of too many breeding. Another example is the United States. It is said that a Asian fish bred too fiercely, and people engaged in electric shock fishing. Of course, these measures are aimed at maintaining the balance of ecology (between species). In the big family of mathematics and even physics, each branch of the subjects is unified and interdependent, and there is no distinction between high and low. As the matter of fact, Hilbert emphasized the integrity of mathematics in his century report as early as 1900, his 23 problems were basically mathematics, but also included physics. In the 1960s, when Professor Luo-Geng Hua started classes at University of Science and Technology of China, he advocated a one-train teaching method and explicitly opposed the separation of various parts of mathematics. These facts all show the high degree of unity of mathematics and even parts of physics. Our basic belief is that: each subject must respect and help each other in order to achieve

harmonious and healthy development.

## References

- [1] Editorial board for Pao-Lu Hsu (Bao-Lu Xu) Memorial Collection (2010). *Moral articles shows a great example in the world — Commemorating the 100th birthday of Professor Xu Bao-Lu* (In Chinese). Peking University Press.
- [2] Borovkov, K.A. (1994). *Russian — English & English — Russian Dictionary on Probability, Statistics, and Combinatorics*. Philadelphia and Moscow. SIAM.
- [3] Chen, M.F. (1986). *Some new developments in probability theory* (In Chinese). The comprehensive report of the 50th anniversary annual meeting in the Chinese Mathematical Society. Chinese Quarterly Journal of Mathematics 1(1): 104–117.
- [4] Chen, M.F. (1997). *Mathematical problems of stochastic systems* (In Chinese). <http://math0.bnu.edu.cn/~chenmf/files/SciPoul/3.pdf>.
- [5] Chen, M.F. (2004). *From Markov Chains to Nonequilibrium Particle Systems*, 2<sup>nd</sup> ed. World Sci, Singapore.
- [6] Chen, M.F. (2005a). *Eigenvalues, Inequalities, and Ergodic Theory*. Springer, London.
- [7] Chen, M.F. (2005b). *Some interaction of probability theory and other subjects* (In Chinese). Advances in Mathematics 34(6): 661–672; reprint (2013): Math Media 37(4): 16–32.
- [8] Chen, M.F. (2017). *Progress of probability theory* (In Chinese). Chinese Journal of Applied Probability and Statistics 33(5): 538–550.
- [9] Chen, M.F. (2020). *Interpretation of cross-disciplinary research* (In Chinese). Chinese Journal of Applied Probability and Statistics 36(1): 86–110.
- [10] Chen, M.F. (2021). *A new mathematical perspective of quantum mechanics* (In Chinese). Advances in Mathematics 50(3): 321–334.
- [11] Working Group on Mathematical Software (2000). Mathematical Software Working Group Archive (4 texts. 1996 — 2000) (In Chinese). <http://math0.bnu.edu.cn/~chenmf/files/SciPoul/29.rar>.
- [12] Feller, W. (1968). *An Introduction to Probability Theory and Its Applications*. Vol. I, 3rd ed. New York, John Wiley & Sons, Inc. Probability Theory and Its Applications, translated by Di-He Hu, People’s Posts and Telecommunications Press, 2014.
- [13] Feller, W. (1971). *An Introduction to Probability Theory and Its Applications*. Vol. II, 2nd ed. New York, John Wiley & Sons, Inc. Probability Theory and Its Applications, translated by Yuan-Lu Zheng, People’s Posts and Telecommunications Press, 2008.
- [14] Ministry of Science and Technology (1956). *Outline of the long-term plan for the development of science and technology from 1956 to 1967 (draft amendment)* (In Chinese). [http://www.most.gov.cn/ztlz/gjzcgqy/zcgylshg/200508/t20050831\\_24440.htm](http://www.most.gov.cn/ztlz/gjzcgqy/zcgylshg/200508/t20050831_24440.htm).
- [15] Editorial board for New English-Chinese Dictionary of Mathematics (2002). *New English-Chinese Mathematical Vocabulary*. Science Press, Beijing.
- [16] Kolmogorov, A.N. (1933). *Foundations of the Theory of Probability*. German version 1933, English translation 1st ed. 1950, 2nd ed. 1956. Chinese version of “Basic Concepts of Probability Theory” (1952), translated by Shou-Ren Wang.

[17] Wang, Z.K. (1965). *Stochastic Processes Theory* (In Chinese). Science Press.

**Acknowledgement:** This study was supported by the National Natural Science Foundation of China (Project No.: 12090011, 11771046), National key R & D plan (No. 2020YFA0712900). Supported by the “double first class” construction project of the Ministry of education (Beijing Normal University) and the advantageous discipline construction project of Jiangsu Universities. The content of this paper was reported at Wuhan University (2021-5-14): the author thanks professors Zhao Huijiang, Zhang Xicheng and Gao Qingqing for their invitation and funding. At the same time, I thank Professor Guo Xianping for his valuable suggestions, which improved the quality of this paper. This article was also reported at the international conference “the 16th Workshop on Markov processes and related topics” (July 12-16, 2021. Changsha, Hunan).

**Mailing Address:**

1 Research Institute of Mathematical Science, Jiangsu Normal University. Shanghai Lu 101, Tongshan Xinqu, Xuzhou, Postcode: 221116.

2 Key Laboratory of Mathematics and Complex System, School of Mathematical Sciences, Beijing Normal University. Beijing, Postcode: 100875.

**E-mail:** mfchen at bnu dot edu dot cn    **Home Page:** <http://math0.bnu.edu.cn/~chenmf>

Translated from Chinese by Ting Yang. June 25, 2021